THE PROBLEM OF TIME-STREAM EVALUATION: PRESENT VALUE VERSUS INTERNAL RATE OF RETURN RULES

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Our interest in the particular problem of time-stream evaluation for the purpose of making investment decisions has developed from a wider interest in cost-benefit analysis.[1] Although the argument set out below is developed in terms of a government 'agency' (this term covers both departments and public corporations or nationalized industries), it is quite general and might be used equally well in the private sector. For simplicity, many of the most interesting problems in the cost-benefit field, such as the measurement of benefits, are ignored in this paper.

This note is divided into three parts: in the first we define the two concepts we are to compare, and consider their significance and usefulness in a world in which there is no crude allocation of investment funds to agencies, but merely a standard laid down which must be met by any investment project. This section ends with a simple example which should illustrate the superiority of the present value procedure. The second part carries the argument into the field of 'sub-optimisation' where funds are allocated to agencies in fixed quantities and it is their task to decide, given the funds, how best to distribute them between different projects. Different projects often involve non-comparable benefits, the benefits of sewers and schools, for example. The cases in which benefits are comparable are typically choices between two ways of doing the same thing: road or rail, dieselisation or electrification, nuclear or conventional power. This is the problem of choosing between incompatible projects and is considered in some detail in each of the first two parts.

In the third part we tie up some loose ends from the previous discussion: this is only done for the case of the present value rule as the problems we consider there are extremely difficult, if not impossible, to handle by any internal rate of return rule.

I. Two Discounting Rules for Time-Stream Evaluation

Two discounting procedures have commonly been proposed for evaluating independent[2] investment projects where the investing agency is not subjected to budget constraints and faces a single lending-borrowing rate of interest.

The present value of a proposed investment is calculated by discounting[3] the

[1] We are indebted to P. D. Henderson, D. L. Munby, J. F. Wright, R. J. Van Noorden and other members of a seminar that discusses this field at Oxford.

[2] By 'independent' we mean that the revenues (social benefits) and costs of one project are independent of whether or not, or on what scale, any other project is undertaken. Where two or more projects are significantly interdependent every possible combination of them must be evaluated separately and the rules for incompatible projects, developed below, applied to choosing between them.

[3] This discounting need not be by any constant rate; the rate applied between years t and t+1 can be different from that applied between t+1 and t+2.

net revenue (social benefit) stream over the life of the project. The net revenue is defined, as for a cash flow, to be net only of actual outgoings,[1] whether on current or on capital account: no separate allowance for depreciation is made. Thus typically the initial capital investment makes the value of the net revenue heavily negative in the early years. In the absence of a budget constraint any project with a positive present value is admissible. Where two incompatible projects (for example the same project on different scales), are both admissible that with the higher present value should be undertaken. A special case of incompatibility is between doing an investment now and doing it at some future date. It may be that the present value for future implementation is the higher in this case our rule requires that the project be postponed.

The internal rate of return of a project is defined as that discount rate which makes the present value of the net revenue (benefit) stream equal to zero. If this rate if greater than the borrowing rate facing the agency, or some arbitrary rate[2] handed down from above, the project is admissible. For choosing between incompatible projects it is often suggested that the alternative with the highest internal rate of return should be chosen. This rule is incorrect because a larger project may have a lower internal rate of return then a smaller one but still have a rate of the difference of the outlays which exceeds the minimum required.[3] In fact it is very difficult to lay down any general criterion for size, but the problem could be avoided by using Fisher's[4] 'rate of return over cost' rule. The rate of return over cost is defined, where two projects are being compared, as that rate of discount which equates the stream of *differences* between the net revenues (benefits) of the two investments to zero.[5] Then of two admissible but incompatible projects, A and B, A should be chosen in preference to B if the internal rate of return of the stream of differences, A–B, is greater than the minimum rate required.[6]

This procedure is equivalent to examining the incremental rate of return; it is the analogue, in the discontinuous case, of considering the marginal internal rate of the project in the continuous case. Although Fisher's incremental rule is based on internal rate of return notions, it escapes from the difficulty that there is no marginal internal rate of return in the discontinuous case. Therefore the typical discontinuity of public sector investment projects is not, in principle, a reason for preferring the present value rule.[7] There are, however, other reasons

[1] For social purposes, both outgoings and revenues should be valued at shadow prices that reflect their social costs.

[2] This arbitrary rate may be some normative rate reflecting a government decision on the time preference to be used for planning purposes.

[3] When this minimum is the market rate of interest it does, of course, represent the opportunity cost of the finance, so that when the incremental rate of return exceeds the minimum it represents a better investment than the next best alternative use of the funds it requires.

[4] I. Fisher, *The Theory of Interest*, New York, Macmillan, 1930 p. 155 in Kelley's reprint of 1960. See also A. A. Alchian, 'The Rate of Interest, Fisher's Rate of Return over Cost', and Keynes' Internal Rate of Return', *American Economic Review*, Vol. xlv (Dec. 1955), p. 938.

[5] It also is the rate at which the two projects have the same present value.

[6] Certain further conditions must also be met if the internal rate of return is to be unique and useful; we discuss these below.

[7] Thus, we cannot agree with Ralph Turvey's emphasis on project discontinuity as a reason for rejecting the use of the internal rate of return. R. Turvey 'Present Value versus Internal Rate of Return—an Essay in the Theory of the Third Best', *Economic Journal*, Vol. lxxiii (March 1963), especially pp. 94 and 98.

for regarding it as superior. First, the value of the internal rate of return need not be unique[1] or even real. Only if the net revenue steam changes sign once only, from negative to positive, it is the case that there must be a unique internal rate and that it is a reasonable thing to 'maximise'. If it changes sign once, but in the opposite direction, a unique rate will exist but the larger it is the less desirable the project will be on present value grounds; for the higher the future cost of some present gain the higher the rate of discount needed to reduce them to the same value. If the net revenue stream changes sign n times there may possibly be up to n distinct values of the internal rate of return.[2] Consider the following example taken from Hirshleifer:[3] —1, +5,—6. This can be discounted to zero at either 100 per cent or 200 per cent. In another example of Hirshleifer's —2, 6, —5 the rate is imaginary. $(1 \pm \sqrt{-1})/2$. All the rates will be imaginary if the present value of the revenue stream is either positive at all discount rates or negative at all discount rates.

It might be argued that these points are not very telling as the examples given are rather peculiar. This is perfectly true if they are regarded as time streams of the net revenue of a project;[4] but as we have already seen a valid rule based on the internal rate of return must also require the use of Fisher's rate of return over cost. In this case time streams which change their sign relatively frequently, and which have non-unique internal rates of return, are far more likely. The circumstances that give rise to these cases also require that the present value rule be applied with precision. For it follows that given a certain time stream its present value may be positive at two levels of the rate of discount and negative in between. This has two implications. first, it is not necessarily true that a project which is admissible at a high rate of discount will be at a lower rate Second, if ambiguity is to be avoided the discount rate must be precisely specified. For instance if the discount rate is meant to reflect 'social time preference' one might be tempted to put limits on it and say that at least one can be sure that what is acceptable at the upper limit is admissible; but this is not true. If social time preference is really lower in the range it might well require that the project be rejected.

Even if a single real value can be attached both to the simple and the incre-

[1] See the discussion in J H Lorie and L J Savage, 'Three Problems in the Rationing of Capital', *Journal of Business* (Chicago) 1955 Pitchford and Hagger, 'A Note on the Marginal Efficiency of Capital', *Economic Journal*, Vol lxviii (1958), (p 597) and comments by Hirshleifer (p 592), Silcock (p 816), Soper (p 174) and Wright (p 813) in the *Economic Journal*, Vol lxix (1959), J Hirshleifer, 'On the Theory of the Optimal Investment Decision', *Journal of Political Economy*, Vol 66 (1958) (p 329), and J F Wright, 'Notes on the Marginal Efficiency of Capital', *Oxford Economic Papers*, N S, Vol xv (1963), (p. 329)

[2] If the net revenue is initially negative (so that the project represents an investment) it is a sufficient, but not a necessary, condition for there to be more than one positive real rate of return (if there is one at all) that there be some point in the time stream such that the sum of the undiscounted net revenues beyond that point is negative

[3] 'On the Theory of the Optimal Investment Decision'

[4] Even the peculiar case is not as rare as some of the contributors to the discussion of multiple rates would have us believe Not all terminal expenses are avoidable, as they seem to assume. Unavoidable costs are most likely to arise in the private sector where there is legislation on the state in which works can be left Thus if open-cast mine works and gravel pits have to be recovered and the topsoil replaced, the terminal expenses may be very high; similarly with the decontamination of abandoned nuclear installations. In evaluating *social* benefits and costs, these negative terminal values exist even if there is no legislative 'internalisation' of them, social valuation takes such externalities into account.

mental rates of return the comparison of them with any current interest rate may be irrelevant if the rate is liable to change over the life of the project. In the case of a present value calculation, on the other hand, one is not committed to using the same rate of discount throughout. One is entirely free to apply any time-preference function without restricting choice to the special case in which it can be represented as a constant discount rate.

Finally, it is far easier to compute and compare the present values of incompatible projects than to calculate Fisher's rates of return over cost for a large set of projects. If we want to evaluate combinations of independent projects[1] it is much simpler to add present values than to recalculate the internal rates of return of the overall time streams. The present value of two independent projects taken together is the sum of their separate present values. No such simple rule can be devised for combining rates of return.

Consider this example of the time streams of two incompatible projects (which have both been selected to have unique internal rates of return)

stream A: —100, 2, 10, 15, 20, 25, 35, 30, 30.
stream B: —100, 5, 15, 25, 30, 25, 20, 20, 20.
stream A–B: 0, −3, −5, −10, −10, 0, 15, 10, 10.

The internal rate of return of A is 10 per cent, that of B 11 per cent, the simple rate of return rule would have us choose B. But if we look at Fisher's rate of return over cost we find that it is 6 per cent for (A–B) and (B–A). As the stream (A–B) changes sign from negative to positive it represents a profitable investment at any interest rate less than 6 per cent, only at rates above 6 per cent would (B–A) be profitable. Thus on Fisher's rule one would choose A if the minimum value of the rate of return were less than 6 per cent and B if it were more. This is exactly the same as the present value rule for Fisher's rate of return over cost is that rate which equates the present values of the two projects. At 5 per cent A has present value 29.0 and B 27.6 at 9 per cent the ordering is reversed with A at 4.2 and B at 6.7.

This example should make clear three important points first, that the simple rate of return rule is wrong: second, that where Fisher's rate of return rule gives any meaningful result, it is the same as the present value rule with a constant discount rate equal to the minimum value in the internal rate of return rule. third, the most substantive objection to Fisher's rule is that in some cases non-uniqueness arises.

II. Discounting Rules under Capital Rationing

The argument of the previous section was set in the context of an unconstrained capital budget. That is to say one in which, while it may be that agencies are told that their projects must have a certain minimum internal rate of return or a positive present value at a certain rate of discount, if this condition is met they can invest as much as they like. This is not the way the world in fact works, governments, especially, like to decide how much is to be invested and by whom, unless they know in advance all about every possible project this control cannot be exercised simply by adjusting the minimum requirements. Thus the constraint

[1] We shall consider in section III several reasons why we might

often becomes a simple limitation on capital expenditure,[1] usually only specified for some short period ahead but in fact being repeated indefinitely. The problem is on what criterion, under these circumstances, projects should be compared and selected.

We shall continue to compare the present value and internal rate of return as providing alternative bases for rules in this situation. We shall consider the budget constraint as a rigid limit either on investment expenditure or on borrowing.

If one calculates the present-value-per-current-pound[2] ratio for each of the projects, where 'current-pounds' stands for the amount of expenditure (borrowing) required in the period of the specified constraint, one can rank the projects in an order of desirability and work down the list until the budget is exhausted. One should cast this rule into an incremental form, as a small project with a high value of the critical ratio should not be allowed to displace a larger[3] one unless the sum of its present value and that of the new project admitted to take up the available funds is greater than that of the larger project. If the present-value-per-current-pound ratio for the border-line project is called the 'marginal' value of the ratio, a larger project is to be preferred to a smaller one with a higher value of the ratio so long as the incremental value of the ratio for the larger project is greater than the marginal value.

Again, under capital rationing, an otherwise admissible project may gain by postponement; then a different project, with a lower ratio of present value to current outlay than the postponed project would have had for immediate implementation, should be undertaken. But future budgets may also be constrained and the marginal value of the ratio may be higher in the future. Thus even if its present-value-per-(then)current-pound ratio would be increased by postponement it would not necessarily be correct to postpone the project if in the same time the marginal ratio rose by a larger amount.[4]

The possibility that future budgets may also be constrained has other complicating implications. For if any part of the revenue of the project accrues to the agency and influences the amount the agency can then invest (e.g. because the constraint is only on borrowing), a value reflecting the positive present value of the (then) marginal project must be assigned to this part of the revenue.[5]

In the case of adapting the internal rate of return so as to provide a possible rule for suboptimisation, one line of approach is to try to find the rate of interest which would, on the usual internal rate of return rule, make admissible expendi-

[1] We assume, though it is perhaps inconsistent, that the government will not restrict recurrent expenditure on projects that have once been accepted. Thus it is only capital and not total expenditure that is constrained. This assumption does not rule out the possibility that the size of current, or prospective, outlays on current account may influence the size of the funds allotted for investment. This case is considered in section III.

[2] This criterion is developed in Hirshleifer *et al.*, Water Supply, Economics, Technology, and Policy, Chicago 1960, Appendix to Chapter VII

[3] Where there is a budget constraint of the type we are considering, projects can be ranked for size by the amount of the constrained resource that they require

[4] A theoretical linear programming approach to this complex postponement problem may be found in S. A. Marglin, *Approaches to Dynamic Investment Planning*, Amsterdam North-Holland (1963).

[5] This is gone into more fully in section III below.

ture exactly equal to the permitted budget. McKean[1] reaches a similar criterion by trying to find a set of projects which will exhaust the budget and also be such that at some discount rate each project has a positive present value while each of the rejected projects either has a negative present value or is incompatible with an accepted project. The problem is, given two or more incompatible projects, how one should rank them; the rule, as stated, does not provide any answer. McKean suggests ranking them by their present values when discounted at the internal rate of return of the marginal admissible project; but this procedure is difficult where projects are of different sizes so that the value of the marginal rate itself depends on which of the projects is selected. Admittedly it is probable that the difference between these rates of return would not be sufficient to reverse the ordering of the projects, but nonetheless it is an unsatisfactory situation.

When the budget was unconstrained, despite all the other disadvantages of the internal rate of return, it was at least possible that its critical value should reflect some chosen (e.g. social) time preference. It is only by the purest chance that it should do so when the budget is constrained. On the other hand, the marginal-present-value-per-current-pound rule continues, even when one is suboptimising, to make a chosen time-preference function an explicit factor in the investment decision. Similarly, the use of McKean's marginal internal rate of return rule precludes explicit inclusion of the social opportunity cost of funds transferred from private investment in future years.

III. Some Complications in the Application of the Present Value Rule

Those who say that in choosing between incompatible projects one should prefer the project with the higher internal rate of return often do so on some assumption that the revenues can be reinvested at that higher rate. Our argument would be that if any *change* in investment can be associated with a project then it should be regarded as part of it and the present value of the whole taken into account. Thus for our purposes the present value of a project reflects the fruitfulness of any further investment it occasions, *but only if there is some special reason for associating the future investment with the present project*.

A possible example could arise where doing one project rather than another would provide an opportunity for a 'better than marginal' investment. For example a demand might exist at a fixed price for a certain number of units of electricity; this could be met either by building a conventional or a nuclear power station, the former would last forty, and the latter twenty, years; the capital and running costs differ in each case, the latter being relatively low for nuclear power. Which should one do? Should one simply apply the present value rule to the problem as it stands?

The selection of the longer lived conventional station, would, assuming demand to be constant and output the same, preclude the possibility of building any power station in twenty years time (except in the unlikely case where the total costs of the new one were less than the variable costs of the old), but it is almost certain that technical progress would so change the cost level and

[1] R. N. McKean, *Efficiency in Government through Systems Analysis*, New York, Wiley, 1958

structure that, at the same price, there would, after twenty years, be the opportunity for a very worthwhile investment. In such a case, the present value, at the time of taking the first decision, of this later station should be added to that of the shorter lived nuclear power station when its relative merits are considered. Taking the present value of a specific future investment into account is justified only where one of a pair of incompatible projects does, and the other does not, preclude the exploitation of some specific future opportunity.

If the discount rate used in the calculation of present values based perhaps on some notion of 'social time preference', is lower than the 'rate of return' on marginal investment in the rest of the economy, any change[1] in aggregate investment attributable to the project, whenever undertaken, should be valued at its present value at the time of the original decision and added to that of the project.[2] A similar procedure is required where the projects influence the amount of investment that can be undertaken by the agency in future periods; this investment will often have a non-zero present value when the agency budget is constrained, even if the discount rate used is not less than the return on investment in general.

[1] Where the change is negative the value of the investment forgone is a part of the opportunity cost of the project.

[2] This idea is discussed and a method of calculating the present value of a change in private investment is developed in M. S. Feldstein, 'Net Social Benefit Calculations and the Public Investment Decision', *Oxford Economic Papers*, March 1964.

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